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***x bar* and *R* Charts**

Statistical process control is an effective method for improving a firm's quality and productivity. There has been an increased interest in their effective implementation in American industry, brought about by increased competition and improvements in quality in foreign-made products. Many tools may be utilized to gain the desired information on a firm's quality and productivity. Some of the more commonly used tools are control charts, which are useful in determining any changes in process performance. These include a variety of charts such as *p* charts, *c* charts and *x bar* and *R* charts. In this paper, I will be focusing on the latter two mentioned.

X bar Charts defined

An *x bar* chart is used to monitor the average value, or mean, of a process over time. For each subgroup, the *x bar* value is plotted. The upper and lower control limits define the range of inherent variation in the subgroup means when the process is in control.

R Chart defined

An *R* Chart is a control chart that is used to monitor process variation when the variable of interest is a quantitative measure. Now, what does all this mean? These charts will allow us to see any deviations from desired limits within the quality process and, in effect, allow the firm to make necessary adjustments to improve quality.

The Chart Construction Process

In order to construct *x bar* and *R* charts, we must first find our upper- and lower-control limits. This is done by utilizing the following formulae:

$$UCL = \mu + \frac{3\sigma}{\sqrt{n}}$$

$$LCL = \mu - \frac{3\sigma}{\sqrt{n}}$$

While theoretically possible, since we do not know either the population process mean or standard deviation, these formulas cannot be used directly and both must be estimated from the process itself. First, the *R* chart is constructed. If the *R* chart validates that the process variation is in statistical control, the *x bar* chart is constructed.

Steps in Constructing an R chart

1. Select k successive subgroups where k is at least 20, in which there are n measurements in each subgroup. Typically n is between 1 and 9. 3, 4, or 5 measurements per subgroup is quite common.
2. Find the range of each subgroup R(i) where R(i)=biggest value - smallest value for each subgroup i.
3. Find the centerline for the R chart, denoted by

$$RBAR = \frac{1}{k}\sum R(i)$$

4. Find the UCL and LCL with the following formulas: UCL= D(4)RBAR and LCL=D(3)RBAR with D(3) and D(4) can be found in the following table:

Table of D(3) and D(4)

n	D(3)	D(4)	n	D(3)	D(4)
2	0	3.267	6	0	2.004
3	0	2.574	7	.076	1.924
4	0	2.282	8	.136	1.864
5	0	2.114	9	.184	1.816

5. Plot the subgroup data and determine if the process is in statistical control. If not, determine the reason for the assignable cause, eliminate it, and the subgroup(s) and repeat the previous 3 steps. Do NOT eliminate subgroups with points out of range for which assignable causes cannot be found.
6. Once the R chart is in a state of statistical control and the centerline RBAR can be considered a reliable estimate of the range, the process standard deviation can be estimated using:

$$\hat{\sigma} = \frac{RBAR}{d(2)}$$

d(2) can be found in the following table:

n	d(2)	n	d(2)
2	1.128	6	2.534
3	1.693	7	2.704
4	2.059	8	2.847
5	2.326	9	2.970

Steps in Constructing the XBAR Chart

1. Find the mean of each subgroup XBAR(1), XBAR(2), XBAR(3)... XBAR(k) and the grand mean of all subgroups using:

$$\bar{\bar{x}} = \frac{1}{k}\sum XBAR(i)$$

2. Find the UCL and LCL using the following equations:

$$UCL = \bar{\bar{x}} + A(2)RBAR$$
$$LCL = \bar{\bar{x}} + A(2)RBAR$$

A(2) can be found in the following table:

n	A(2)	n	A(2)
2	1.880	6	.483
3	1.023	7	.419
4	.729	8	.373
5	.577	9	.337

3. Plot the LCL, UCL, centerline, and subgroup means
4. Interpret the data using the following guidelines to determine if the process is in control:

a. one point outside the 3 sigma control limits
b. eight successive points on the same side of the centerline
c. six successive points that increase or decrease
d. two out of three points that are on the same side of the centerline, both at a distance exceeding 2 sigma's from the centerline
e. four out of five points that are on the same side of the centerline, four at a distance exceeding 1 sigma from the centerline
f. using an average run length (ARL) for determining process anomalies

Example:

The following data consists of 20 sets of three measurements of the diameter of an engine shaft.

n	meas#1	meas#2	meas#3	Range	XBAR
1	2.0000	1.9998	2.0002	0.0004	2.0000
2	1.9998	2.0003	2.0002	0.0005	2.0001
3	1.9998	2.0001	2.0005	0.0007	2.0001
4	1.9997	2.0000	2.0004	0.0007	2.0000
5	2.0003	2.0003	2.0002	0.0001	2.0003
6	2.0004	2.0003	2.0000	0.0004	2.0002
7	1.9998	1.9998	1.9998	0.0000	1.9998
8	2.0000	2.0001	2.0001	0.0001	2.0001
9	2.0005	2.0000	1.9999	0.0006	2.0001
10	1.9995	1.9998	2.0001	0.0006	1.9998
11	2.0002	1.9999	2.0001	0.0003	2.0001
12	2.0002	1.9998	2.0005	0.0007	2.0002
13	2.0000	2.0001	1.9998	0.0003	2.0000
14	2.0000	2.0002	2.0004	0.0004	2.0002
15	1.9994	2.0001	1.9996	0.0007	1.9997
16	1.9999	2.0003	1.9993	0.0010	1.9998
17	2.0002	1.9998	2.0004	0.0006	2.0001
18	2.0000	2.0001	2.0001	0.0001	2.0001
19	1.9997	1.9994	1.9998	0.0004	1.9996
20	2.0003	2.0007	1.9999	0.0008	2.0003

RBAR CHART LIMITS:

RBAR = 0.0005

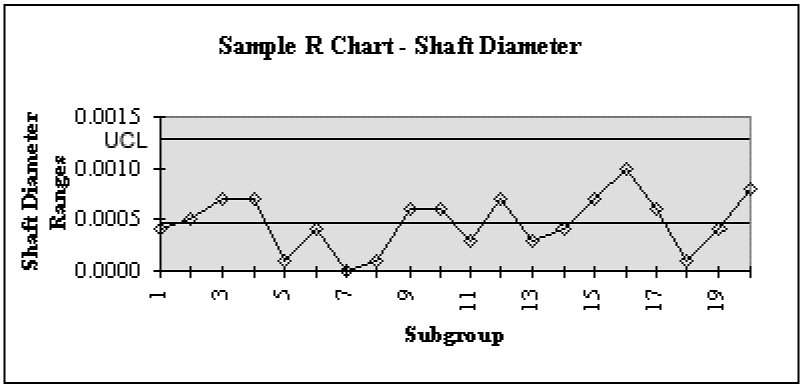
UCL=D(4)*RBAR = 2.574 * .0005 = 0.001287
LCL=D(3)*RBAR = 0.000 * .0005 = 0.000

XBAR CHART LIMITS:

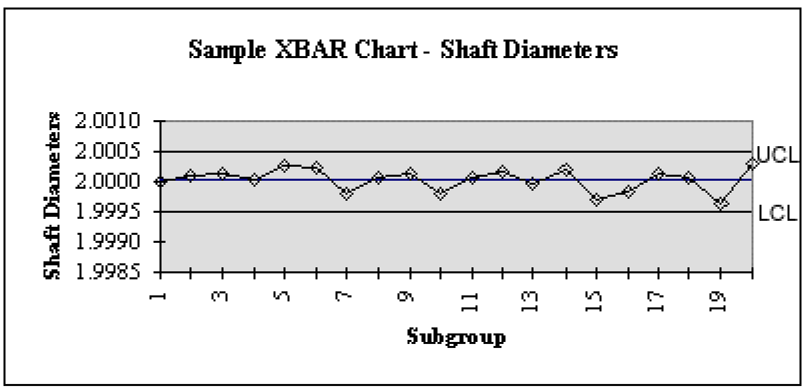
XDBLBAR = 2.0000

UCL = XDBLBAR + A(2)*RBAR = 2.000+1.023*.0005 = 2.0005115
LCL = XDBLBAR - A(2)*RBAR = 2.000-1.023*.0005 = 1.9994885

R - Chart:



XBAR - Chart:



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