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***P*-Charts: The Proportion Defective Attribute Quality Control Charts**

Quality has been a concept that has been with us since the beginning of time. From the creation of the world as described in the book of Genesis in the Bible, “God pronounced his creation ‘good’—e.g., acceptable quality” (Systma). From that point on, quality has been an issue that cannot be easily defined. However, certain intrinsic elements or dimensions of quality have become more manageable and more importantly, controllable. One such element is the *p*-chart, or a proportion defective attribute control chart.

With the advent of Statistical Process Control (SPC), a new methodology process created an ongoing and continuously evolving process rather than a quick-fix quality management system. SPC uses statistics to identify quality variations in a process. Steps are then designed and implemented to correct these variations, bringing processes under ever-tighter control resulting in higher quality. With SPC, companies can monitor their processes throughout production, analyze data and take corrective action in real-time. With this monitoring, quality assurance can be determined and adhered to.

There are two principal types of Quality Assurance: “sampling inspection of incoming outgoing materials (acceptance sampling) and control charts for ongoing processes (process control). Control charts differentiate between the process being in control (within an accept range of random variation) and out of control (outside the acceptable range)” (Antony 9). The primary purpose of a control chart is to detect whether a major change or shift is imminent or has occurred in a process resulting in an alteration of that process. The selection of the appropriate control chart is vital for the success of the quality of process.

Attribute Control charts are used when gathered data or measurements are classified as acceptable or not acceptable (pass/fail, go/no go, good/bad). These attributes provide counting data. This data or actual number of failures or the fraction, or percent of failures is charted (Smith 189). The *p*-chart is the most commonly used attribute chart. The p represents the fraction, or percent, of the number of items that are unacceptable (or defective). The *p*-chart is most helpful in monitoring and controlling the percentage of defective parts in a production run

(Amsden 75). It tracks the fraction of nonconforming items in a sample run. Samples may consist of consecutive items taken at a specified time according to a random sample plan or 100 percent of the production for a specified time period (hour, day, week). The acceptable/non-acceptable decision of the process may be based on one characteristic or several, but a defective (non-acceptable) item is counted as defective only once, even though it may have several defects (Smith 191).

Prior to actual conducting the *p*-chart, preparation must occur to ensure the accuracy of the intended data. The process must be clearly defined. Quality control teams can be effective in developing flow charts and cause and effect diagrams for the process analysis. The characteristics that will be managed or measured need also be defined. This could be as simple as go-no go gauge at a specific measurement point or something more difficult as a judgment call of pass/fail. When individual judgment is used for deciding product conformity, inspectors must all be consistent and in total agreement as to what exactly is classified as defective (Smith 192).

The procedures for creating a p-chart can be segmented into six basic steps. The first is to gather all needed data, select the size, frequency and number of samples. The sample size should be large enough to ensure that most of the samples will have a nonzero number of defects. The sample size should be big enough to give an average of five or more defectives per sample. A sample size of at least 50 units is a good starting point. The calculations and interpretation of the chart are easier when the sample size is kept constant. When constant sample times are used and the sample size varies, a single set of control limits can be used as long as the sample sizes do not differ by more than 25 percent of the average sample size. If the sample sizes differ by more than 25 percent, separate control limit calculations will be needed (Amsden 85).

Sample frequency should factor in the production schedule. The *p*-chart should give an accurate depiction as possible of the process at the specific point in question. Every item produced in that time should have an equal chance of being chosen in a sample (Smith 192). The number of actual samples should be at least 20 samples. The data collection timeframe should be long enough to track all possible sources of variation (Smith 193).

The second step in constructing a *p*-chart is calculating the *p* (percentage) for each sample. This is accomplished by dividing the number of defects (*np*) by the actual sample size (*n*) [$p = np/n$]. If the number of defects in the sample of 50 units were 4, the *p* would equal .08 [$p = 4/50$; .08).

The third step in the *p*-chart is setting the scale for the chart or graph. After several *p* values have been calculated, make the scale from 0 to approximately twice the largest know *p* value. This will allow for all the *p* values to fit on the graph (Smith 193).

The fourth step is to plot all *p* values and connect all plot points with a straight line (Amsden 89).

Calculating the *p*-bar and control limits is the fifth step to the *p*-chart. The *p*-bar is calculated by adding all *np*'s (total number of defects) and dividing by the total amount of units in all samples. If the total number of defects was 134 units, and the total number of samples was 20 with 50 units in each sample (1000 total units), the p-bar value would be .134 or 134/1000. A solid line should be added to the chart to indicate the *p*-bar value.

Calculating the control limits can be quite difficult and tricky. However, with a calculator and some patience, the control limits can be determined. The control limits have multiple calculations so precision is critical for the proper results. The exact formula is as follows:

$$UCL = pbar + 3\sqrt{\frac{pbar(1-pbar)}{n(i)}}$$

$$LCL = pbar - 3\sqrt{\frac{pbar(1-pbar)}{n(i)}}$$

The Upper control limit (UCL) will be the first calculation to consider. From the example numbers listed above while calculating the *p*-bar, it can be determined that the UCL in this case would be .2785. The Lower control limit (LCL) would calculate to a -.0105, and since the answer is negative, the LCL is set at zero. A broken line should be added to the chart to indicate both the UCL and the LCL. It is a good idea to check all the steps and ensure that the arithmetic is correct (Amsden 91). The following are the completed calculations for control limits:

UCL	= .134 + 3√.134 x (1. - .134)/50	LCL	= .134 - 3√.134 x (1. - .134)/50
	= .134 +3√.134 x .866/50		= .134 -3√.134 x .866/50
	= .134 +3√.1164044/50		= .134 -3√.1164044/50
	= .134 +3√.00232088		= .134 -3√.00232088

= .134 +3 (.4818) = .134 -3 (.4818)

= .134 + .1445 = .134 - .1445

= .2785 = -.1050

The last step in consideration with the *p*-chart is to interpret the chart. There are three possible situations to consider. First, all the *p* values are inside the control limits. If this is the case, this can be indicative that the process is in statistical control. Also only inherent variations will be at work with the process.

The second situation to consider is that one or two *p* values are outside the control limits. The usual practice is to not include these values or any sample information from these samples. Also all calculations will need to be refigured including the *p*-bars and control limits. The chart then should be re-examined. If any *p* value is out of the control limits, the process is not in statistical control. The process and causes will need to be determined and resolved. The *p*-chart process should begin again to ensure that the process has been fixed (Amsden 95).

If three or more *p* values are outside the control limits, the process is clearly out of control. Causes must be determined immediately to resolve such issues. After that the data collection can be again.

Shifts of seven or more points to a higher or lower levels may represent that the process has been affected somehow or that the exact cause of the defect has somewhat changed. Trends of a run of seven consecutive points either up or down indicates that something is causing the defects to change in a gradual manner (Smith 194). This type of pattern indicates that something irregular is affecting the process. The causes could be anything from poorly trained personnel to inconsistent inspections. If this occurs separate p-charts may need to take place at other points in the process.

P-charts as well as other factors of SPC have been widely accepted among "quality practitioners as an aid for monitoring, managing, analyzing, and improving process performance by eliminating causes of variations" (Antony 7). The use of this type of data permits a scientific management style in which decisions are made "based on facts, rather than guesswork, and better products can be produced with less scrap and rework" (Fine).

There is a tremendous amount of additional information on *p*-charts as well as the other attribute control charts. I would highly recommend the book, *SPC Simplified: Practical Steps To Quality*, by Amsden, Butler and Amsden. This book instructs the basics of SPC for a general audience. It is directed to everyone from the front line worker on the assembly line to the manager in the back office. A variety of other information can be found on the Internet using most browsers with a search engine and entering the subjects of "Quality Control", "P charts", and/or "Control Charts".

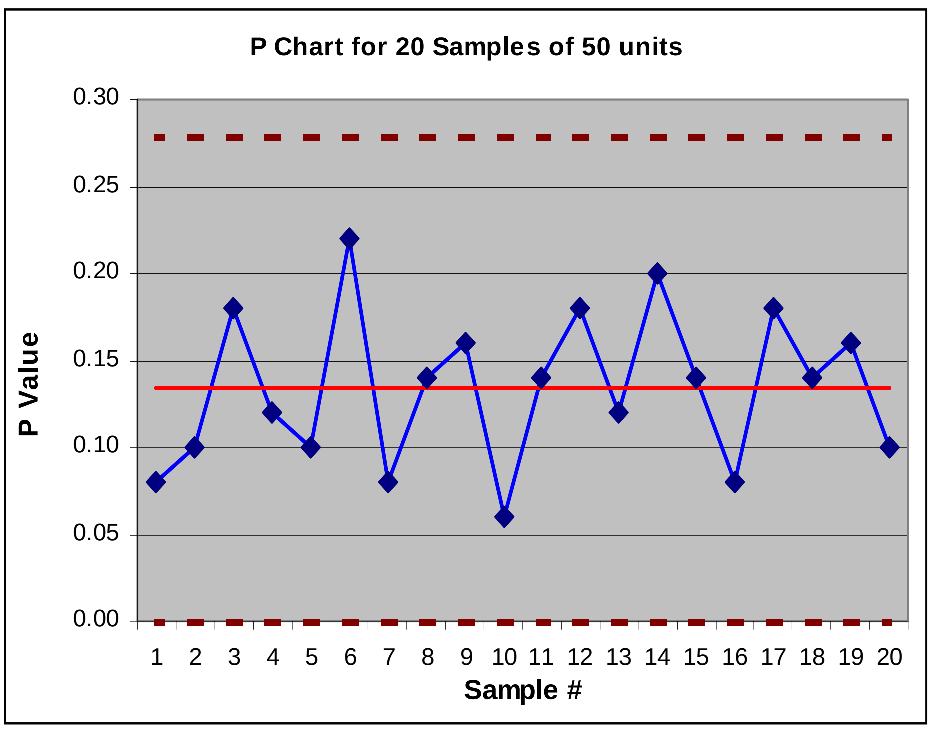
A practical example:

The following is a typical example for conducting a *p*-chart.

Sample	# of defects	*p*-value
n	*np*	*np/n*
50	4	0.08
50	5	0.10
50	9	0.18
50	6	0.12
50	5	0.10
50	11	0.22
50	4	0.08
50	7	0.14
50	8	0.16
50	3	0.06
50	7	0.14
50	9	0.18
50	6	0.12
50	10	0.20
50	7	0.14
50	4	0.08
50	9	0.18
50	7	0.14
50	8	0.16
50	5	0.10
1000	134	0.1340

UCL= .134 + 3√.134 x (1. - .134)/50
= .134 +3√.134 x .866/50
= .134 +3√.1164044/50
= .134 +3√.00232088
= .134 +3 (.4818)
= .134 + .1445
UCL= .2785

LCL= .134 - 3√.134 x (1. - .134)/50
= .134 -3√.134 x .866/50
= .134 -3√.1164044/50
= .134 -3√.00232088
= .134 -3 (.4818)
= .134 - .1445
LCL= -.1050



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